AGGREGATE POPULATION FORECASTING WITH THE USE OF DEMOGRAPHIC POTENTIALS' TECHNIQUE

Dalkhat M. Ediev, Ph.D.

Karachay-Cherkess State Technological Institute, Russia

Aggregate models for population forecasting are constructed and tested on historical data from France, Japan, Sweden, and the US. Russian population forecasts are used in testing as well. Models are based on the concept of demographic potential that is close to the Fishers' reproductive value. Although linear and aggregate, the constructed models show high performance and adequately reflect effects of age-sex structure changes. Aggregate nature of the models facilitates their use in the population forecasting as well as in the demographic and economic modeling, within both short and long time horizons.

Introduction

The method of aggregate economic-demographic modeling based on the demographic potential technique was earlier proposed and tested by author (Ediev 2000). It was tested on data from Sweden and showed high performance in modeling different population indexes, including population size and different age-sex groups. This work more deeply investigates the properties of the method in forecasting the population size. In addition to Swedish data set historical data from three other countries are brought to analysis as well. Besides, known Russian population forecasts till 2020 are used to test forecasting capabilities of approach proposed. More attention is given to effect of horizon length to forecasting accuracy. Comparative study of different projecting alternatives approved high performance of the method proposed and allowed to derive practical recommendations on population forecasting in different horizons.

Population forecasting based on demographic potential concept

Idea of demographic potential inherent to current population state arose and has been exploited in demography since long ago. It can be found in works of R. A. Fisher on reproductive value (Fisher 1930), P. Vincent's and J. Bourgeois-Pichat's works on growth potential (Vincent 1945, Bourgeois-Pichat 1968, 1971, Andreev, Pirozhkov 1975), L. Goodman's work on eventual reproductive value (Goodman 1968), N. Keyfitz's works on stable equivalent and momentum (Keyfitz 1968, 1969, 1971, 1985, Keyfitz, Flieger 1968, 1990) with its numerous extensions (Frauenthal 1975, Mitra 1976, 1987, Cerone 1996; Tuljapurkar, Li 199?, Potter, Wolowyna, Kulkarni 1977, Schoen, Kim 1991), K. Tognetti's concept of gross reproductive worth (Tognetti 1976), and works of L. Hersch and of his successors on potential demography (Hersch 1944).

The concept of demographic potential (Ediev 1999) is related to concepts listed above and arises from different needs of population modeling. It can be constructed as an index proportional to future ultimate size of population posterity. Old person has a small demographic potential since all his children have already been born, and no future posterity can be assigned to oldster. Application of this approach to one-sex stable population model leads to the Fishers' reproductive value (Fisher 1930). Fisher considered birth as a loan given to the child and that child's reproductive value as present value of future debt payments (i.e. his children). Keyfitz examined this concept and showed that reproductive value is proportional to ultimate size of posterity (Keyfitz 1985). Demographic potential concept can be applied to more sophisticated models that incorporate migration, parity-progression, etc. (Ediev 1999). Another approach to demographic potential relates to its properties as aggregate population index. The most important property for population modeling is that rate of demographic potential change equals to asymptotic rate of population increase (Lotkas' intrinsic growth rate). This property is sufficient to construct the demographic potential (Ediev 1999). Most simple realization of the demographic potential concept is within the framework of one-sex population model. In this case demographic potential for person at age *x* is given by formula (Ediev 1999):

$$c(x) = \frac{e^{rx}}{l(x)} \int_x^\infty l(y) f(y) e^{-ry} dy, \qquad (1)$$

here *r* is intrinsic growth rate, $l(x), f(x)$ - survivorship and fertility functions. Hereinafter variables *x* and *y* will be used to denote ages, *t* – as time variable. As well as in the previous work (Ediev 2000) age-specific demographic potentials used here are calculated on 1991 US vital data and are presented in table 1.

Table 1. Demographic potentials for the USA, 1991.

Age	Males	Females	Age	Males	Females
0	1.00	1.00	**28**	0.55	0.39
1	1.00	1.00	**29**	0.49	0.33
2	1.01	1.01	**30**	0.43	0.28
3	1.01	1.01	**31**	0.38	0.23
4	1.01	1.01	**32**	0.33	0.19
5	1.01	1.01	**33**	0.28	0.15
6	1.01	1.01	**34**	0.24	0.12
7	1.01	1.01	**35**	0.20	0.09
8	1.01	1.01	**36**	0.16	0.07
9	1.01	1.01	**37**	0.14	0.05
10	1.01	1.01	**38**	0.11	0.04
11	1.01	1.01	**39**	0.09	0.02
12	1.01	1.01	**40**	0.07	0.02
13	1.01	1.01	**41**	0.06	0.01
14	1.01	1.01	**42**	0.05	0.01
15	1.01	1.00	**43**	0.04	0.00
16	1.01	0.99	**44**	0.03	0.00
17	1.00	0.96	**45**	0.02	0.00
18	0.99	0.93	**46**	0.02	0.00
19	0.97	0.89	**47**	0.01	0.00
20	0.94	0.84	**48**	0.01	0.00
21	0.91	0.80	**49**	0.01	0.00
22	0.87	0.74	**50**	0.01	0.00
23	0.82	0.68	**51**	0.01	0.00
24	0.77	0.62	**52**	0.00	0.00
25	0.72	0.56	**53**	0.00	0.00
26	0.66	0.50	**54**	0.00	0.00
27	0.60	0.44	**55**	0.00	0.00

Given the potentials for different age groups (1) and age structure of the population, aggregate demographic potential can be calculated as:

$$C(t)=\int_0^{\infty} n(x;t)c(x)dx\ , \qquad (2)$$

here *C(t)* is total demographic potential, and $n(x;t)$ is population density in exact age *x* at time *t*.

Although potentials (1)-(2) relate to one-sex populations, two-sex generalization can be developed. Following formula is used to obtain two-sex potential:

$$C_{total}(t) = (1+\gamma)\sqrt{C_{female}(t)\cdot C_{male}(t)/\gamma} \tag{3}$$

here $C_{population}(t)$ is aggregate demographic potential of given population at time *t*, γ - is sex ratio at birth (males to females). There is no direct generalization of intrinsic growth rate concept for two-sex model. Taking into account properties of demographic potential of one-sex population, change rate of aggregate demographic potential (3) can be considered as intrinsic growth rate of two-sex population. Given the (projected or modeled) values of intrinsic growth rate one can forecast aggregate demographic potential without age-sex analysis:

$$C(t) = C(t_0)\cdot \exp\left(\int_{t_0}^{t} r(\tau)d\tau\right), \tag{4}$$

here *C(t)* is total demographic potential, *r(t)* – intrinsic growth rate at time *t*. When intrinsic growth rate is constant, demographic potential became exponential function of the time variable *t*.

Using equation (4) – in spite of (2), – we are free from age-sex structure modeling. This equation is the basis for aggregate forecasting of the population.

The size of the population couldn't be adequately modeled in the same simple way as demographic potential. Age structure changes in the population and inertia of demographic processes make such a model too simplistic. Fortunately, the fact that despite its aggregate nature, model (4) adequately reflects age-sex structure changes of the population opens the way to aggregate modeling of the population.

Although the size of population with aging process going on can't be modeled in simple way, and no general equation of its dynamic can be proposed, some reasoning can be made about the ratio of the size $N(t)$ and demographic potential (i.e. the mean demographic potential $c(t)$):

$$c(t) = \frac{C(t)}{N(t)} \tag{5}$$

This ratio is constant for population with stable age-sex structure. For non-stable populations age-sex structure tends to converge to its ultimate stable structure

(Arthur 1982; Schoen, Kim 1991). We can therefore expect that the ratio (5), that reflects current age-sex structure of the population, continually converges to its asymptotic value too. Let’s denote the ultimate value of ratio (5) as $c^*(t)$. Like stable age-sex structure, this value depends on survivorship function and intrinsic growth rate:

$$c^*(t)=\frac{(1+\gamma)\sqrt{\frac{1}{\gamma}\cdot\int_0^\infty l_{females}(x)e^{-r(t)x}c_{females}(x)dx\cdot\int_0^\infty l_{males}(y)e^{-r(t)y}c_{males}(y)dy}}{\int_0^\infty l_{females}(x)e^{-r(t)x}dx+\gamma\int_0^\infty l_{males}(x)e^{-r(t)x}dx} \tag{6}$$

As the ratio (5) tends to converge to its stable equivalent (6), the simplest model reflecting this property, is following:

$$\frac{dc(t)}{dt}=\alpha\cdot\left(c(t)-c^*(t)\right) \tag{7}$$

Discrete analogous of (7) is the following equation:

$$c(t+1)-c(t)=a\cdot\left(c(t)-c^*(t)\right), \tag{8}$$

here *a* can be estimated using common regression procedures.

Dynamic of $c(t+1)-c(t)$ and $c(t)-c^*(t)$ for US female population is presented on fig. 1. Being in general consistent with (8), pattern shown on the figure 1 is characteristic for system with hysteresis, i.e. for situation when $c(t+1)-c(t)$ has an inertia. Model (7) can be adjusted in order to reflect that inertia of $c(t+1)-c(t)$:

$$\frac{d^2c(t)}{dt^2}=\beta\cdot\left(\frac{dc(t)}{dt}-\alpha\cdot\left(c(t)-c^*(t)\right)\right). \tag{9}$$

Correction of discrete relation (8) is straightforward:

$$c(t+1)-c(t)=a\cdot\left(c(t)-c^*(t)\right)+b\cdot\left(c(t)-c(t-1)\right) \tag{10}$$

Performance of (10) can be observed on fig. 2 that shows actual and modeled dynamic of the same parameters as on the fig.1.

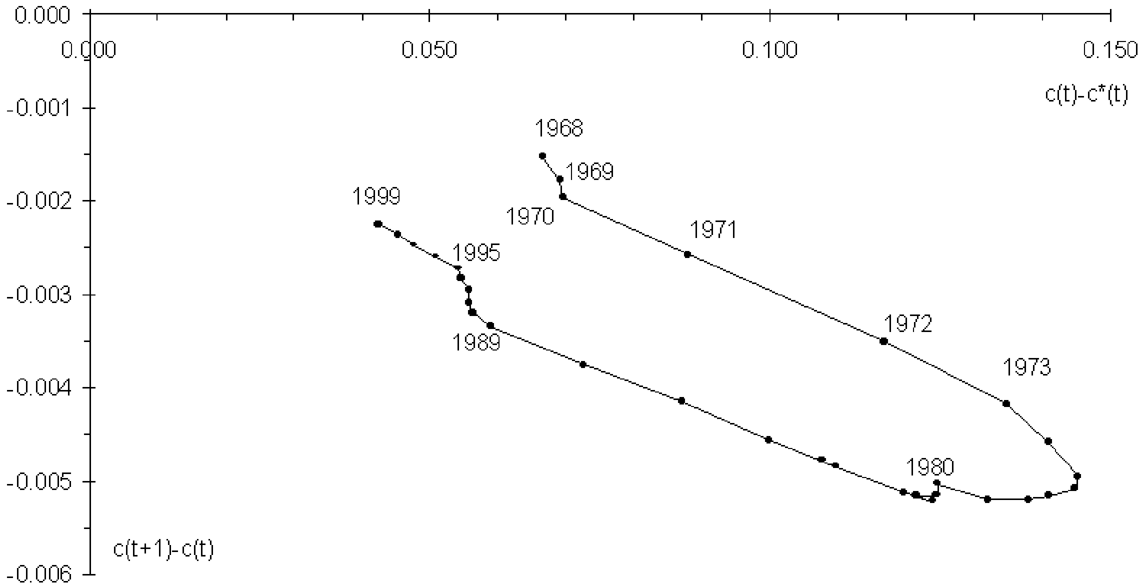

Fig. 1. Changes and distance to the stable equivalent for mean demographic potential of US female population.

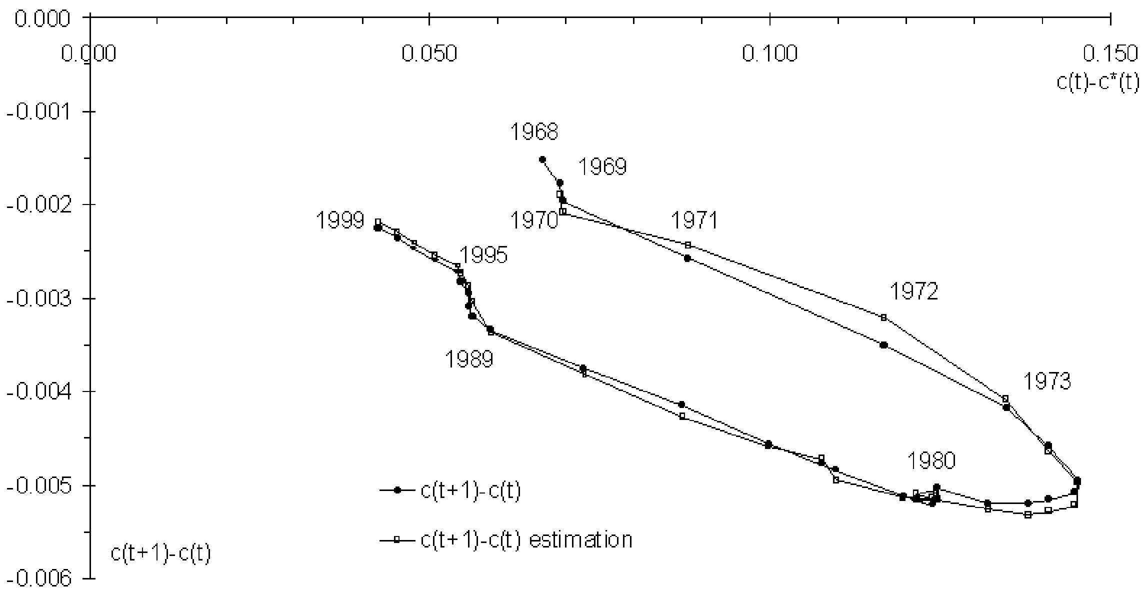

Fig. 2. Changes and distance to the stable equivalent for mean demographic potential of US female population – actual and modeled by (10).

Models (7)-(8) and (9)-(10) together with the model (4) for demographic potential can be used in modeling the population size. Rearranging equation (5) we have:

$$N(t) = \frac{C(t)}{c(t)}. \qquad (11)$$

Model completed by (11) is aggregate except for relations used to estimate stable equivalent of ratio (5), i.e. except for relation (6). The only age-sex structure dependent function of the model - the stable equivalent $c^*(t)$ - can be modeled in aggregate way too. Dynamic of $c^*(t)$ depends on dynamic of two population characteristics – intrinsic growth rate $r(t)$ and survivorship function $l(x;t)$. In order to develop aggregate model, one could replace age-sex dependent survivorship function by some aggregate measure of survivorship. Life expectancy at birth $e_0(t)$ can be used for this purpose:

$$c^*(t) = c \cdot r(t) + d \cdot e_0(t) + const \tag{12}$$

Relation (12) completes aggregate model of population size:

$$\text{continuous form:} \quad \begin{cases} C(t) = C(t_0)\exp\left(\int_{t_0}^{t} r(\tau)d\tau\right), \\ \dfrac{dc(t)}{dt} = \alpha \cdot \left(c(t) - c^*(t)\right), \\ c^*(t) = c \cdot r(t) + d \cdot e_0(t) + const, \\ N(t) = \dfrac{C(t)}{c(t)} \end{cases} \qquad \text{discret form:} \quad \begin{cases} C(t) = C(t_0)\exp\left(\int_{t_0}^{t} r(\tau)d\tau\right), \\ c(t+1) - c(t) = a \cdot \left(c(t) - c^*(t)\right), \\ c^*(t) = c \cdot r(t) + d \cdot e_0(t) + const, \\ N(t) = \dfrac{C(t)}{c(t)} \end{cases} \tag{I}$$

or, using more accurate relation (9) reflecting the inertia of population structure:

$$\text{continuous form:} \quad \begin{cases} C(t) = C(t_0)\exp\left(\int_{t_0}^{t} r(\tau)d\tau\right), \\ \dfrac{d^2c(t)}{dt^2} = \beta \cdot \left(\dfrac{dc(t)}{dt} - \alpha \cdot \left(c(t) - c^*(t)\right)\right), \\ c^*(t) = c \cdot r(t) + d \cdot e_0(t) + const, \\ N(t) = \dfrac{C(t)}{c(t)} \end{cases} \qquad \text{discret form:} \quad \begin{cases} C(t) = C(t_0)\exp\left(\int_{t_0}^{t} r(\tau)d\tau\right), \\ c(t+1) - c(t) = \\ = a\left(c(t) - c^*(t)\right) + b\left(c(t) - c(t-1)\right), \\ c^*(t) = c \cdot r(t) + d \cdot e_0(t) + const, \\ N(t) = \dfrac{C(t)}{c(t)} \end{cases} \tag{II}$$

External parameters that should be specified in order to apply constructed models to given population are intrinsic growth rate $r(t)$, life expectancy at birth $e_0(t)$, the value of demographic potential $C(t)$ at any point of time, and beginning value of $c(t)$ for model (I), or the value and derivative of $c(t)$ at the beginning of time horizon – for (II). All these parameters except for first two ones, should be taken from the age-sex structure of real (or modeled) population. Intrinsic growth rate and life expectancy are to be modeled separately (e.g. as dependents of economic variables). For open populations some arrangements are to be made concerning migration.

Testing different aggregate models on empirical data

Performance of models presented above in population forecasting is tested on long-range historical data from France, Japan, Sweden, and the US (data were taken from the Berkeley Mortality Database; University of California 1998). In addition, Russian population forecasts till 2020 were used in testing as well.

In order to refine age-sex structure changes, intrinsic growth rate, life expectancy, and migration are taken equal to actual values.

Quality of model (I) is seen as early as on the parameter estimation stage. Estimation of parameters for equations (10) and (12) was very successful (*a=-0.0126, b=0.683, c=8.86, d=-0.00325, const=0.619*) – almost all determination coefficients were close to 0.9. The same result was obtained when the model was applied to one-sex populations. As for the model (I), equation (10) is to be replaced by (8). Estimation of *a* in that equation (*a=-0.35*) wasn't so successful, determination coefficient was close to 0.9 only for part of the populations analyzed.

Result of population forecasting for countries mentioned above using models (I) and (II) is shown on figs. 3-6 (initial demographic potential and ratio $c(t)$ at initial time points are estimated from actual age-sex structures). It worth to note, that errors of both models are cumulated during projecting period, i.e. during up to 150 years. Both models show good compliance with empirical data, though the model (II) seems to be more adequate.

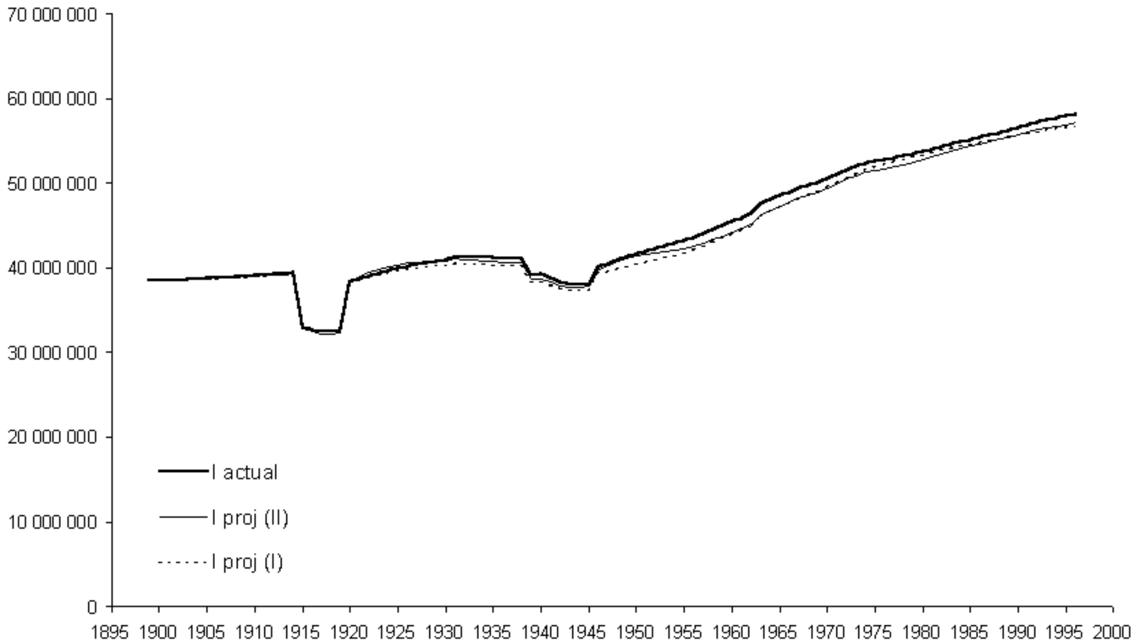

Fig. 3. Actual and projected population of France.

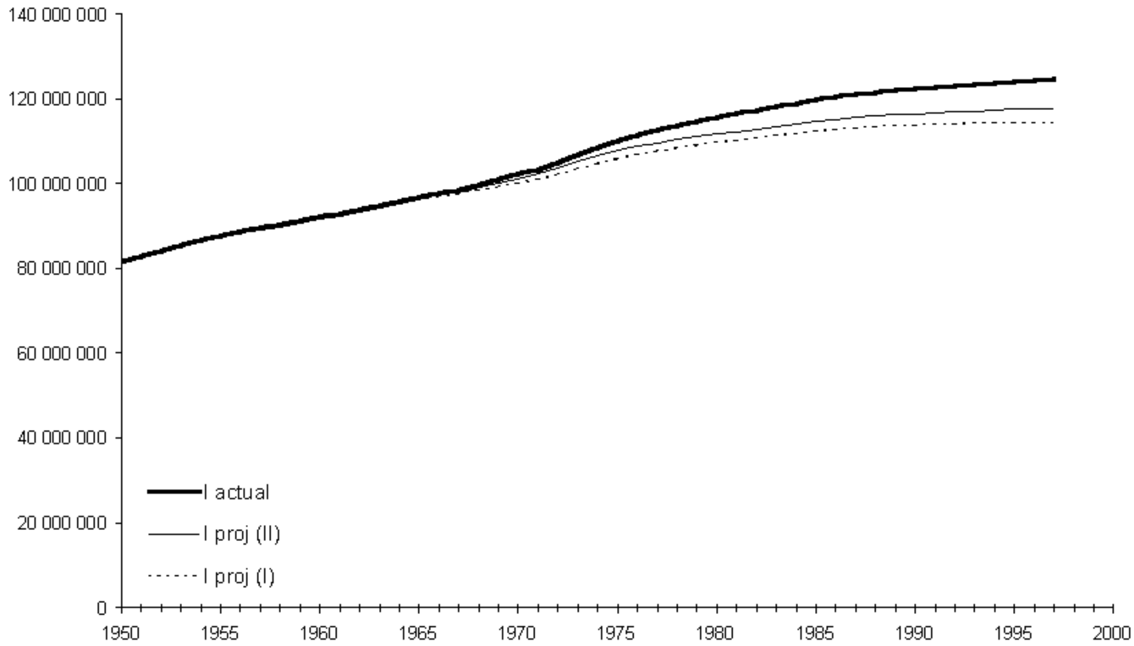

Fig. 4. Actual and projected population of Japan.

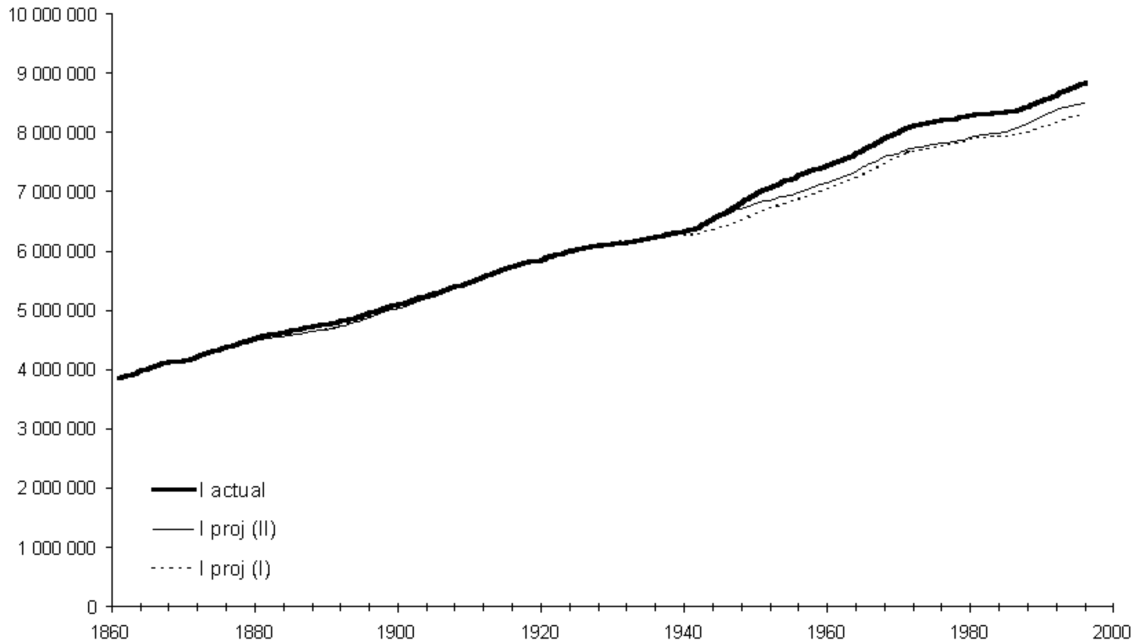

Fig. 5. Actual and projected population of Sweden.

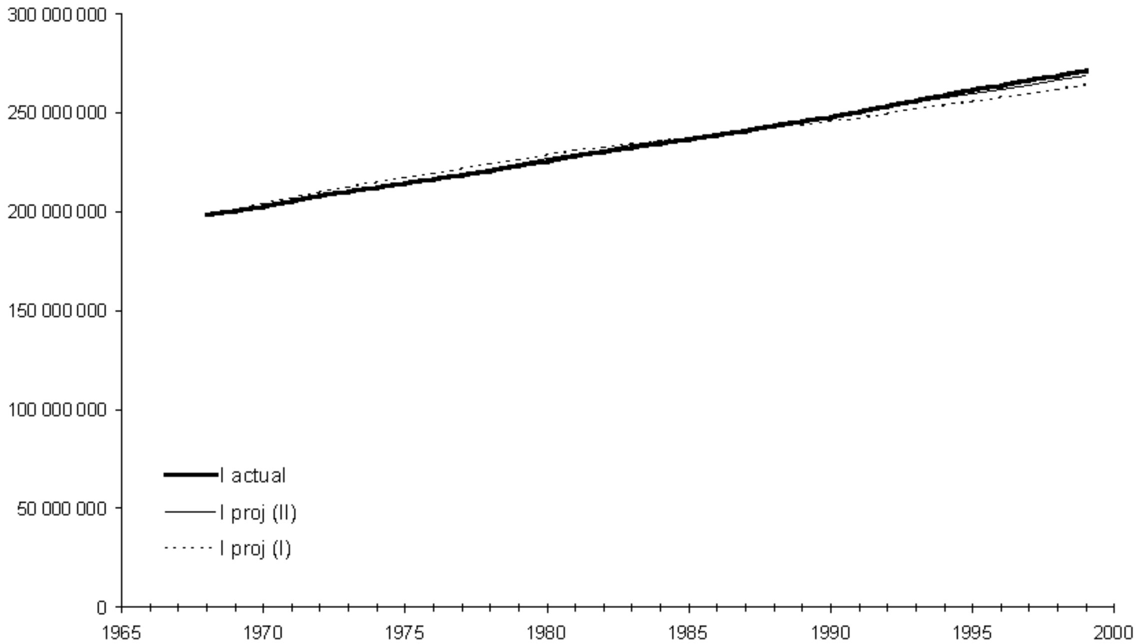

Fig. 6. Actual and projected population of the US.

For comparative purposes different ‘naive’ projecting techniques are used in addition to (I) and (II) with several forecasting horizons.

First alternative forecasting method is linear extrapolation of the population size. Different projecting horizons are used – from 1 to 10 years. In this method data correction for migration isn’t made. This leads to errors increase, especially for countries with remarkable ‘migrational’ changes (data on France correspond to different border lines).

Width of database used to estimate parameters of extrapolation was set to be equal the forecasting horizon plus one year for all methods.

Second forecasting method differs from (I) and (II) in modeling the dynamic of ratio (5) – linear extrapolation is used in spite of (8) or (10). As well as in the previous method, different forecasting horizons and lengths of data set are used in extrapolation.

Third forecasting method ignores the process of aging. In this method population change rate supposed to be equal to intrinsic growth rate. This technique is used with variety of constant forecasting horizons too.

Next forecasting technique is opposite to previous one – the process of aging supposed to be instantaneous, i.e. momentum of population growth set to be one at any time, and $c(t) \equiv c^*(t)$ in spite of (8) or (10). This model isn’t aggregate (stable age structure is to be used). But without lack of accuracy it can be replaced by aggregate approximation, using relations (12) and (11).

Application of models (I), (II) to all data range at once, as it’s shown on figs. 3-6, leads to error cumulating and varying forecasting horizon. For comparative study constant forecasting horizons (of 1 to 10 years length) are used too. In order to keep constant forecasting horizon, different initial values for $c(t)$ are used to obtain forecasts for different years. To project population index in 2000 with horizon 10, for example, actual values of $c(t)$ before 1990 are used to project the index till 2000.

Table 2 summarizes determination coefficients obtained during testing the models on data from countries mentioned. The most accurate model among those presented is the inertial model (II), although the model (I) also performs rather well.

Table 2. Determination coefficients between actual and projected populations.

Forecasting horizon, years	Population			
	FRANCE	**JAPAN**	**SWEDEN**	**USA**
Model (I):				
1	1,000	1,000	1,000	1,000

Forecasting horizon, years	Population			
	FRANCE	**JAPAN**	**SWEDEN**	**USA**
2	1,000	1,000	1,000	1,000
3	0,999	1,000	1,000	0,999
4	0,999	0,999	0,999	0,999
5	0,999	0,999	0,999	0,998
6	0,998	0,999	0,999	0,997
7	0,998	0,999	0,999	0,996
8	0,997	0,998	0,998	0,995
9	0,997	0,998	0,998	0,994
10	0,997	0,998	0,998	0,993
Continuously	0,996	0,993	0,992	0,991
Model (II):				
1	1,000	1,000	1,000	1,000
2	1,000	1,000	1,000	1,000
3	0,999	0,999	1,000	1,000
4	0,999	0,999	0,999	1,000
5	0,999	0,999	0,999	1,000
6	0,999	0,998	0,999	1,000
7	0,998	0,997	0,999	1,000
8	0,998	0,997	0,998	1,000
9	0,998	0,996	0,998	1,000
10	0,998	0,996	0,998	1,000
Continuously	0,997	0,996	0,997	0,999
Population without aging (growth rate equals to intrinsic growth rate):				
1	0,999	1,000	1,000	0,999
2	0,997	0,999	0,999	0,997
3	0,994	0,998	0,998	0,993
4	0,989	0,997	0,997	0,987
5	0,983	0,995	0,995	0,979
6	0,976	0,992	0,993	0,969
7	0,968	0,988	0,990	0,958
8	0,959	0,984	0,988	0,946
9	0,950	0,979	0,985	0,932
10	0,941	0,973	0,981	0,917
Continuously	0,866	0,268	0,693	0,751
Instant aging (age-sex structure instantly reaches stable equivalent):				
-	0,697	0,837	0,806	0,465
Linear extrapolation of the population size:				
1	0,968	1,000	1,000	1,000
2	0,937	0,999	1,000	0,999
3	0,883	0,998	0,999	0,999
4	0,811	0,995	0,999	0,998
5	0,754	0,992	0,997	0,997

Forecasting horizon, years	Population			
	FRANCE	**JAPAN**	**SWEDEN**	**USA**
6	0,723	0,987	0,996	0,997
7	0,699	0,981	0,994	0,997
8	0,678	0,974	0,992	0,998
9	0,681	0,966	0,990	0,998
10	0,688	0,956	0,988	0,999
Linear extrapolation of mean demographic potential *c(t)*:				
1	1,000	1,000	1,000	1,000
2	0,999	1,000	1,000	1,000
3	0,996	0,999	0,999	0,998
4	0,991	0,998	0,998	0,994
5	0,984	0,997	0,996	0,989
6	0,975	0,995	0,993	0,983
7	0,963	0,994	0,989	0,977
8	0,950	0,992	0,984	0,971
9	0,934	0,991	0,977	0,967
10	0,916	0,991	0,970	0,966

Although the determination coefficient has transparent statistical meaning and reflects fitness of the forecast to real data, for the purpose of forecasting estimations for possible forecast errors would be of interest yet. This is of special importance since all the populations considered had upward trend in size. Mean absolute percentage errors (MAPE) are given in table 3 for different forecast considered in the paper.

Table 3. Mean absolute percentage errors of population forecasts – for different models.

Forecasting horizon, years	Population			
	FRANCE	**JAPAN**	**SWEDEN**	**USA**
Model (I):				
1	0,1%	0,3%	0,2%	0,3%
2	0,3%	0,5%	0,3%	0,4%
3	0,4%	0,8%	0,4%	0,6%
4	0,5%	1,0%	0,6%	0,7%
5	0,6%	1,2%	0,7%	0,9%
6	0,6%	1,4%	0,8%	1,0%
7	0,7%	1,6%	0,9%	1,1%
8	0,8%	1,8%	1,0%	1,2%
9	0,8%	1,9%	1,1%	1,3%
10	0,8%	2,1%	1,2%	1,3%

Forecasting horizon, years	Population			
	FRANCE	**JAPAN**	**SWEDEN**	**USA**
Continuously	1,6%	3,5%	2,6%	1,3%
Model (II):				
1	0,1%	0,1%	0,1%	0,1%
2	0,2%	0,2%	0,2%	0,1%
3	0,3%	0,3%	0,3%	0,1%
4	0,4%	0,4%	0,4%	0,1%
5	0,5%	0,5%	0,5%	0,2%
6	0,6%	0,6%	0,6%	0,2%
7	0,6%	0,7%	0,7%	0,2%
8	0,7%	0,8%	0,8%	0,2%
9	0,7%	0,8%	0,8%	0,2%
10	0,7%	0,9%	0,9%	0,2%
Continuously	1,2%	2,4%	1,8%	0,4%
Population without aging (growth rate equals to intrinsic growth rate):				
1	0,4%	1,1%	0,4%	0,7%
2	0,8%	2,2%	0,8%	1,4%
3	1,2%	3,2%	1,1%	2,1%
4	1,6%	4,1%	1,5%	2,8%
5	2,0%	5,1%	1,8%	3,4%
6	2,3%	5,9%	2,1%	4,0%
7	2,6%	6,8%	2,5%	4,6%
8	2,9%	7,6%	2,8%	5,1%
9	3,2%	8,4%	3,1%	5,6%
10	3,5%	9,1%	3,4%	6,1%
Continuously	10,8%	19,9%	11,7%	10,1%
Instant aging (age-sex structure instantly reaches stable equivalent):				
-	14,2%	35,2%	12,3%	27,0%
Linear extrapolation of the population size:				
1	0,2%	0,1%	0,1%	0,1%
2	0,5%	0,3%	0,3%	0,2%
3	0,8%	0,5%	0,4%	0,2%
4	1,2%	0,8%	0,6%	0,3%
5	1,5%	1,0%	0,8%	0,3%
6	1,8%	1,3%	1,0%	0,3%
7	2,2%	1,6%	1,3%	0,3%
8	2,6%	1,8%	1,4%	0,3%
9	3,0%	2,0%	1,6%	0,3%
10	3,3%	2,2%	1,7%	0,2%
Linear extrapolation of mean demographic potential *c(t)*:				
1	0,1%	0,1%	0,1%	0,1%
2	0,3%	0,2%	0,2%	0,2%
3	0,6%	0,3%	0,4%	0,4%
4	1,0%	0,4%	0,7%	0,6%
5	1,4%	0,5%	0,9%	0,8%

Forecasting horizon, years	Population			
	FRANCE	**JAPAN**	**SWEDEN**	**USA**
6	1,8%	0,6%	1,2%	1,0%
7	2,2%	0,7%	1,6%	1,1%
8	2,7%	0,7%	1,9%	1,2%
9	3,0%	0,7%	2,3%	1,2%
10	3,3%	0,8%	2,7%	1,2%

Figure 7 represents relationship between the forecasting horizon and MAPE. It's seen that on average for all the countries mentioned (i.e. with total of about 300 years of forecasting) MAPE of model (II) was less than one percent for horizons less than 10 years. Even for continuously forecasting with errors accumulation, MAPE of the model was slightly above a percent and a half. Simpler model (I) leads to errors 0,5% higher for long range forecasting. Even for 10-uear forecasting horizon linear extrapolation models lead to 1% higher errors than model (I) (i.e. twice the error of inertial model (II)). For longer horizons linear extrapolation will lead to even higher errors, while continuous forecasting seems to be impossible to obtain with the use of linear extrapolation. Between two extrapolation methods simpler seems to be better – extrapolation of population size itself, without intermediate extrapolation of mean demographic potential. This pretty amusing result is an evidence that unreasonable sophistication of the model can lead to even higher errors. Least accurate model among those presented on the fig. 7, is the model without aging (5% for 10 year horizon, and about 12% for continuous forecasting). Even higher errors (18%) were obtained for the model with instant aging.

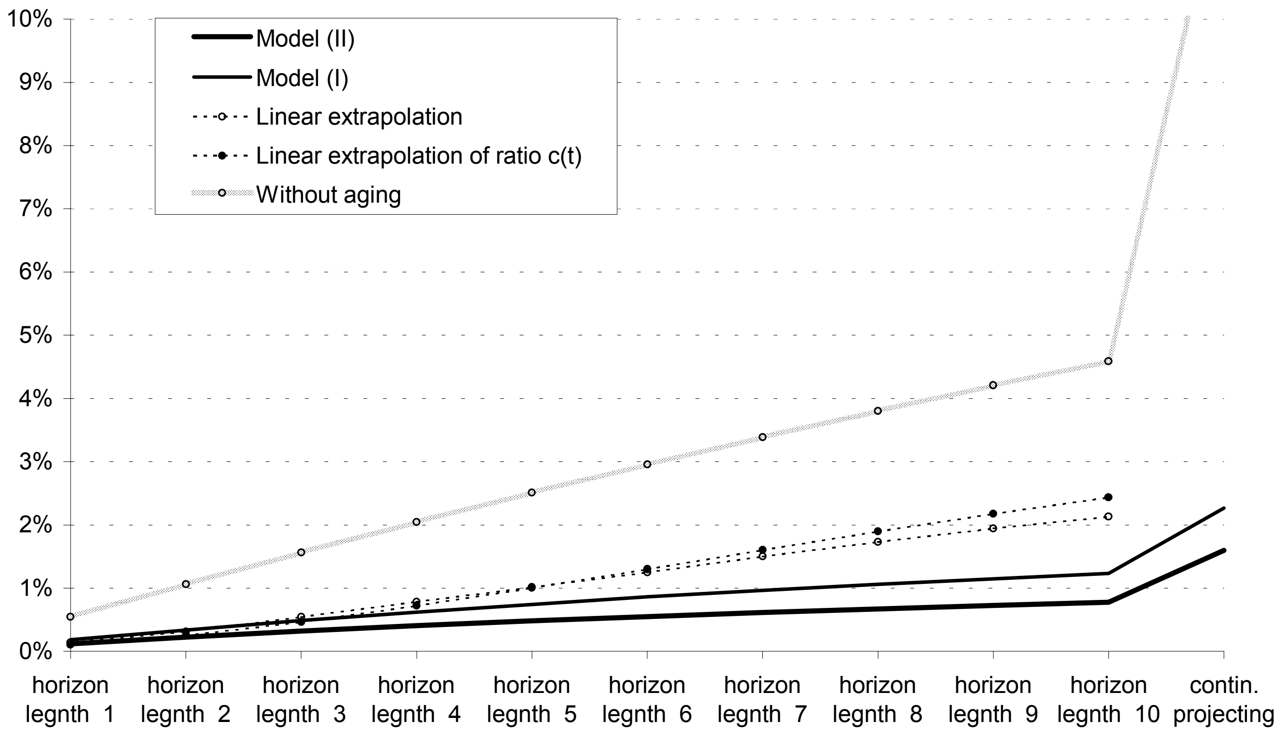

Fig. 7. Mean absolute percentage errors of population forecasts (average for all four populations considered) for different forecasting horizons and models.

Summarizing findings from the analysis of MAPEs, it can be stated that models proposed perform in average rather well. Inertial model (II) performs better than all others in all forecasting horizons, while the linear model (I) is worse than simple linear extrapolation in short-range (less than 2-3 years) forecasting, but it's better in long-range forecasting. This conclusion holds even if the case of France (when extrapolation errors are affected by border changes; corrections made for these changes improved extrapolation errors but didn't eliminate them) is excluded from consideration. When considering the data only from Japan, Sweden, and USA (about 200 years of forecasting) extrapolation errors fall by about 0,5% for 10-year horizon, while performance worsens for the model (I) – by about 0,2%, - and for the model without aging – by 0,5%10-year horizon. As a result, model (I) becomes more preferable than linear extrapolation only in horizons longer than 5-6 years (fig. 8).

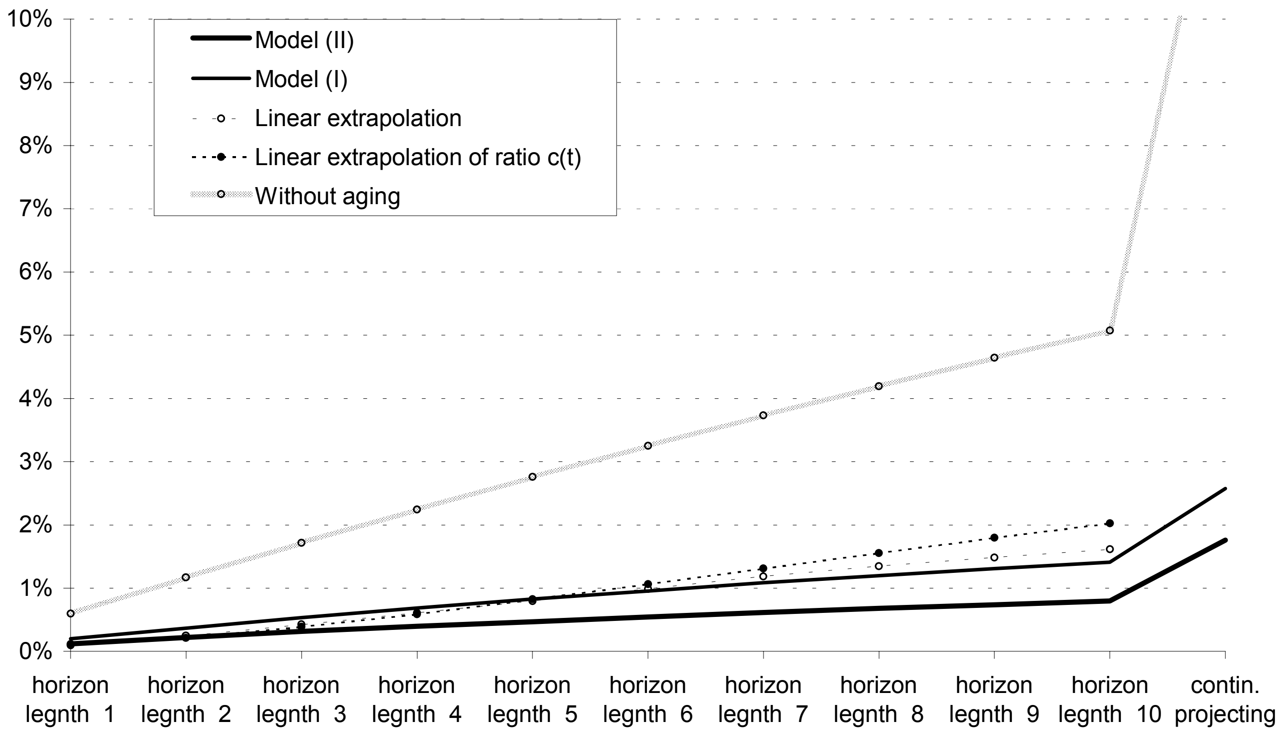

Fig. 8. Mean absolute percentage errors of population forecasts (average for three populations – Japan, Sweden, and the US) for different forecasting horizons and models.

Possible maximum absolute percentage errors (MaxAPE) of forecasts are of interest in addition to MAPEs analyzed before. Table 4 and figs. 9, 10 present these MaxAPEs for forecasts discussed in the paper. MaxAPE of the model (II) is less than 6% even for continuous forecasting, for 10-year horizon it's less than 4%. Linear model (I), lead to higher errors (up to 8,5% for continuous forecasting, and 4,2% - 10 year horizon), but it's much better than linear extrapolation methods. In short (1-3 years) horizons, yet, the model (I), seems to be less accurate than linear extrapolation.

Table 4. Maximum absolute percentage errors of population forecasts.

Forecasting horizon, years	Population			
	FRANCE	**JAPAN**	**SWEDEN**	**USA**
Model (I):				
1	0,5%	0,5%	0,6%	2,3%
2	0,9%	0,9%	1,2%	2,5%
3	1,3%	1,4%	1,8%	2,7%

Forecasting horizon, years	Population			
	FRANCE	**JAPAN**	**SWEDEN**	**USA**
4	1,6%	1,8%	2,3%	2,9%
5	1,8%	2,2%	2,7%	3,1%
6	2,0%	2,5%	3,0%	3,3%
7	2,1%	2,9%	3,3%	3,5%
8	2,2%	3,2%	3,6%	3,7%
9	2,3%	3,5%	3,9%	4,0%
10	2,4%	3,8%	4,1%	4,2%
Continuously	3,4%	8,5%	5,8%	4,9%
Model (II):				
1	0,4%	0,3%	0,5%	0,7%
2	0,7%	0,6%	1,0%	0,8%
3	1,1%	0,9%	1,5%	0,8%
4	1,3%	1,1%	2,0%	0,9%
5	1,6%	1,3%	2,4%	1,0%
6	1,8%	1,5%	2,7%	1,0%
7	1,9%	1,7%	3,1%	1,1%
8	2,1%	1,8%	3,3%	1,1%
9	2,2%	1,9%	3,5%	1,2%
10	2,3%	2,0%	3,7%	1,2%
Continuously	2,8%	5,7%	4,8%	1,9%
Population without aging (growth rate equals to intrinsic growth rate):				
1	1,4%	1,6%	1,3%	1,1%
2	2,6%	3,1%	2,5%	2,1%
3	3,7%	4,5%	3,7%	3,1%
4	4,7%	5,9%	4,8%	4,2%
5	5,4%	7,3%	5,9%	5,1%
6	5,9%	8,6%	6,9%	6,1%
7	6,6%	9,9%	7,9%	7,0%
8	7,5%	11,1%	8,9%	7,9%
9	8,3%	12,3%	9,8%	8,7%
10	9,0%	13,4%	10,7%	9,6%
Continuously	24,0%	41,0%	32,9%	20,5%
Instant aging (age-sex structure instantly reaches stable equivalent):				
-	72,6%	62,3%	52,8%	37,1%
Linear extrapolation of the population size:				
1	1,4%	0,6%	0,7%	0,2%
2	3,0%	0,9%	1,4%	0,6%
3	4,2%	1,4%	2,4%	0,6%
4	6,6%	2,1%	2,7%	0,8%
5	8,0%	2,8%	3,1%	1,0%
6	9,2%	3,5%	3,9%	1,2%
7	10,0%	4,1%	4,7%	1,3%
8	10,6%	4,8%	5,3%	1,4%
9	11,2%	5,6%	6,0%	1,6%

Forecasting horizon, years	Population			
	FRANCE	**JAPAN**	**SWEDEN**	**USA**
10	12,2%	6,6%	6,3%	1,7%
Linear extrapolation of mean demographic potential *c(t)*:				
1	1,8%	0,6%	0,5%	0,2%
2	3,6%	0,6%	1,3%	0,8%
3	5,4%	1,3%	2,2%	1,5%
4	7,2%	2,0%	2,9%	2,4%
5	9,7%	2,6%	4,2%	3,3%
6	10,9%	3,1%	5,5%	4,1%
7	11,4%	3,5%	6,9%	4,6%
8	11,7%	3,6%	8,3%	5,0%
9	12,0%	3,8%	9,9%	5,2%
10	11,9%	3,7%	11,3%	5,2%

Conclusions about models performance can be illustrated graphically. Figs. 11, 12 represent Swedish population forecasting results obtained with the use of different models considered in the paper – for 10-year horizon and for continuous forecasting with errors accumulation.

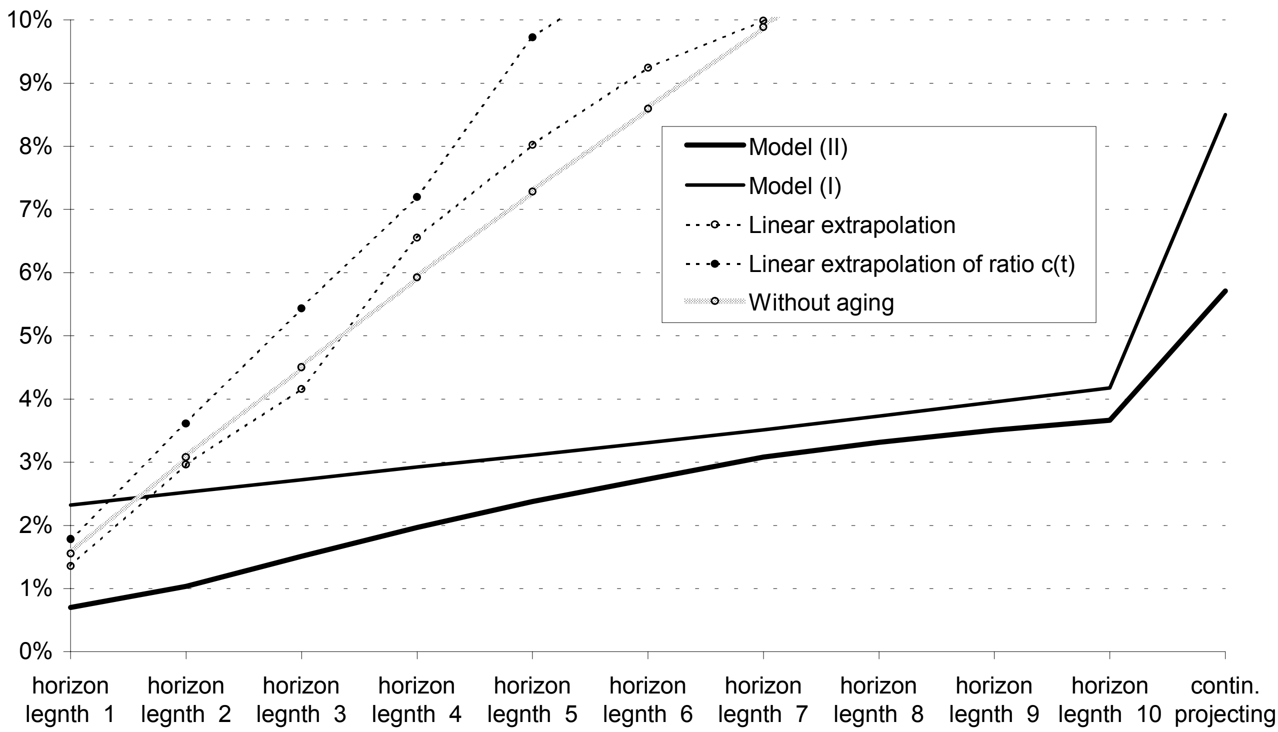

Fig. 9. Maximum (for all four countries) absolute percentage errors of population forecasts for different models and forecasting horizons.

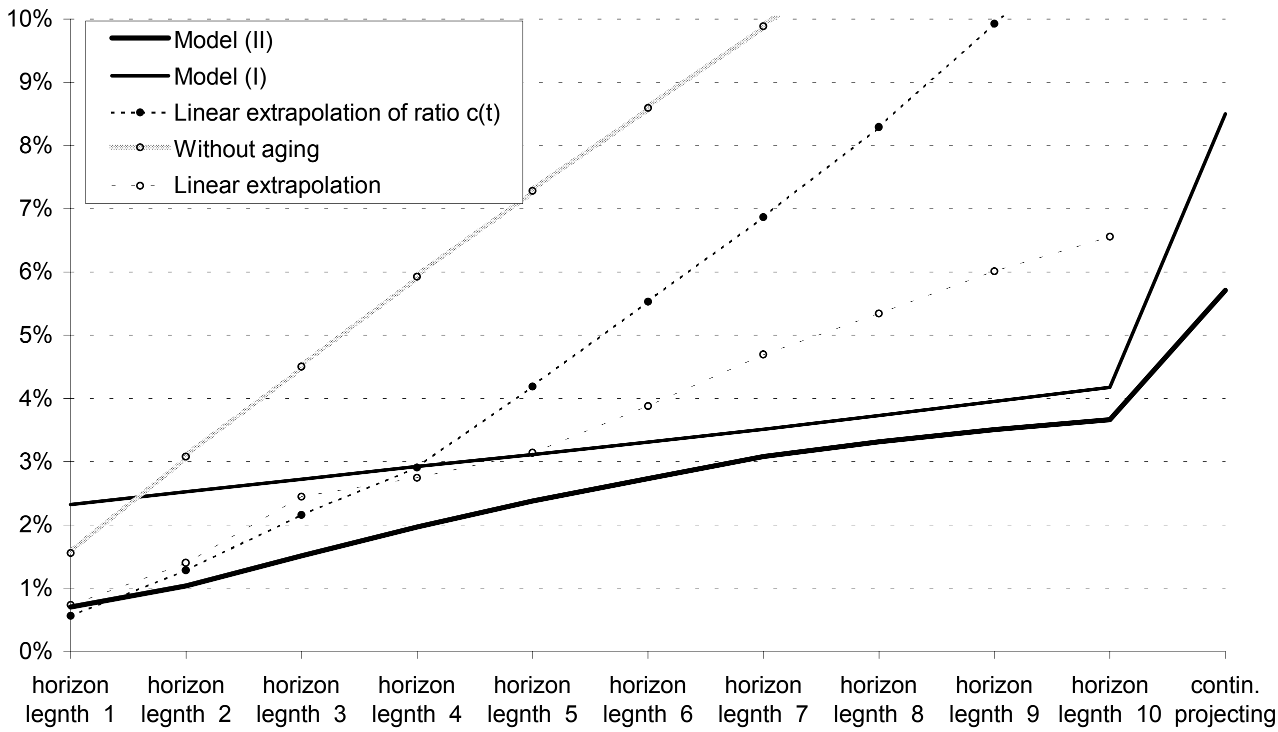

Fig. 10. Maximum (for three countries – Japan, Sweden, and USA) absolute percentage errors of population forecasts for different models and forecasting horizons.

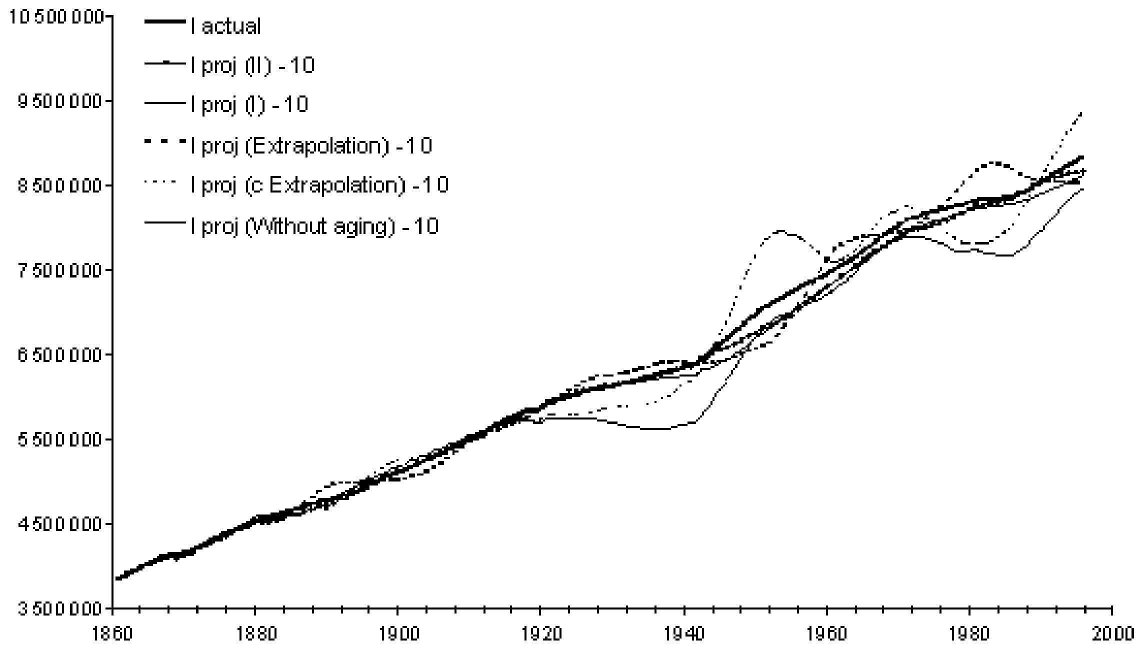

Fig. 11. Actual and projected population size, Sweden, 1861-1996. 10-year forecasting horizon.

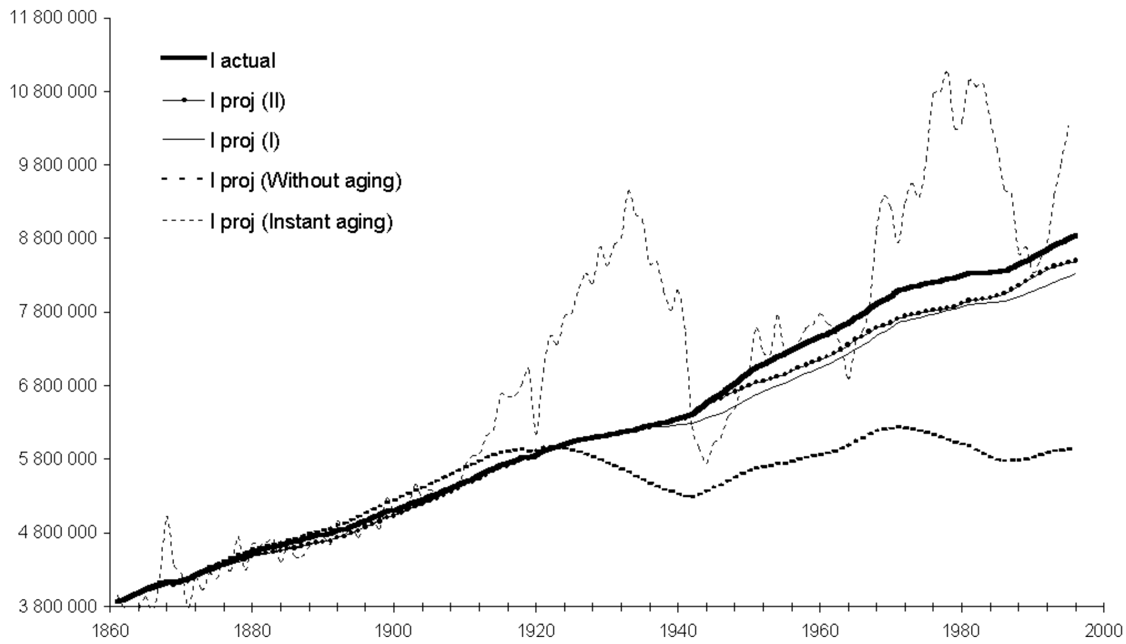

Fig. 12. Actual and projected population of Sweden, 1861-1996 (errors are cumulated since 1861).

In addition to data from the US, France, Sweden, and Japan, perforemance of the model (II) was illustrated on Russian data as well. Unfortunately, author hasn't appropriate long-range data on population structure and mortality in RF. That didn't permit to utilize the same approach as above. In spite of that, model (II) was used to obtain forecasts of Russian population which were compared to analogous forecasts obtained with use ofwell sophisticated multiregional age-sex specific component model (Center for Demography and Humans' Ecology 2000). While getting forecasts from the model (II), same assumptions (on fertility, migration, and mortality) were used as in the component methods' forecasts Center for Demography and Humans' Ecology. Forecast results are presented of the fig. 13. Fig. 14 represents absolute percentage deviations of model (II) forecast from the Center of Demography's forecast for different lengths of forecasting horizon. These results are consistent with those obtained above.

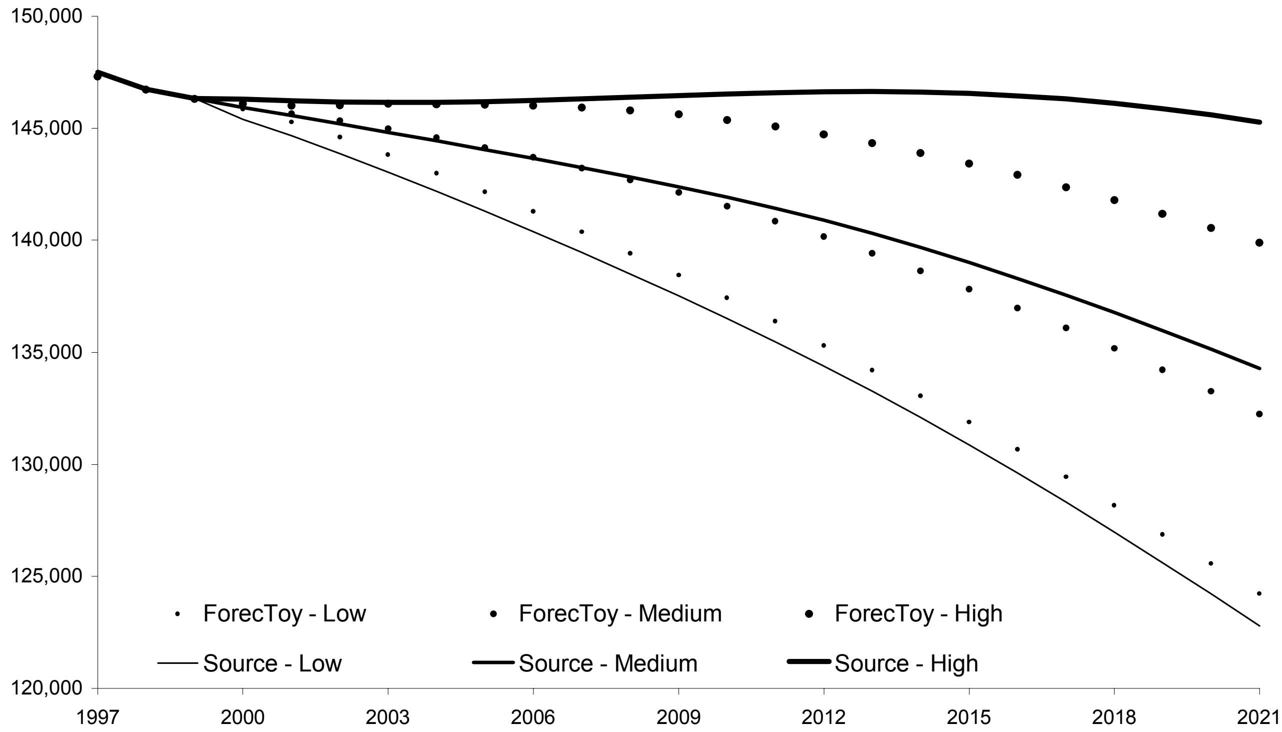

Fig. 13. Russian population forecasts, obtained from the model (II) («ForecToy»), and multiregional age-sex specific component model of Center for Demography and Humans' Ecology («Source»).

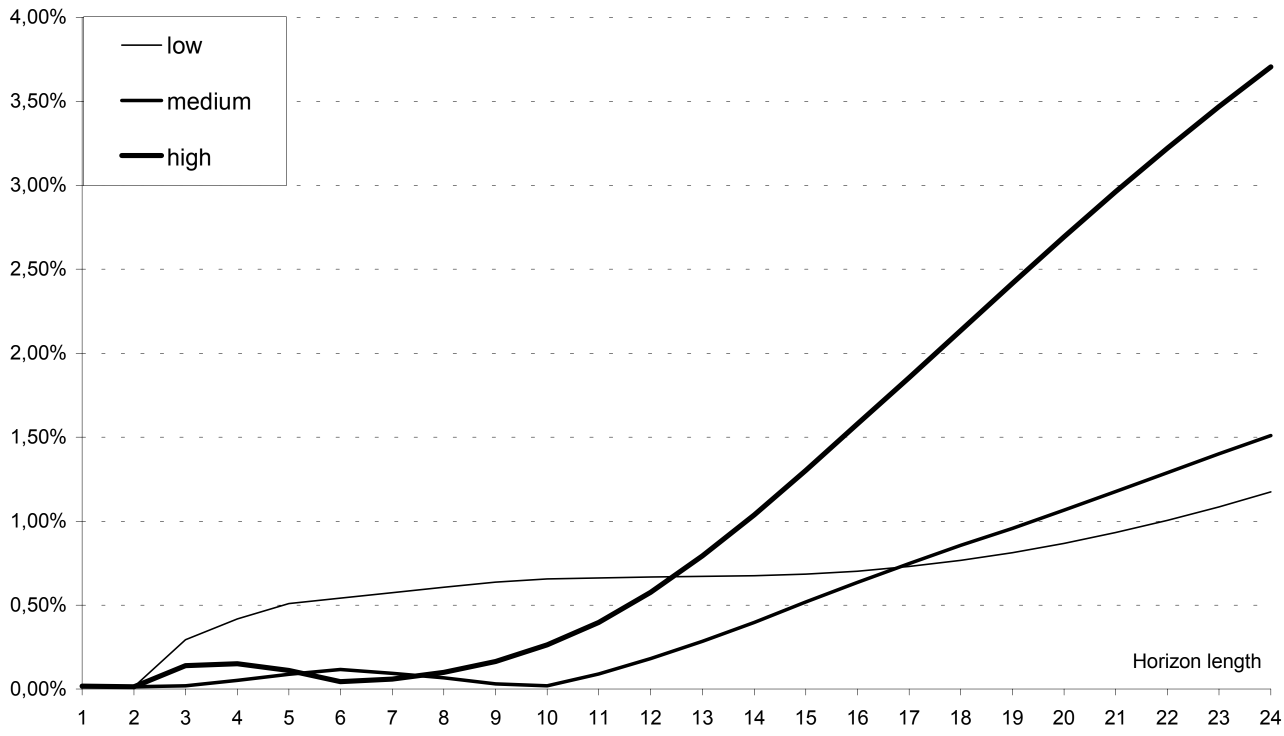

Fig. 14. Absolute percentage deviations of model (II) forecast from those of multiregional age-sex specific component model of Center for Demography and Humans' Ecology.

Summarizing all the findings, it can be stated that models proposed in the paper, especially model (II), are adequate enough to real processes happening in real populations, and can be used in population forecasting. In all the forecasting horizons model (II) leads to less errors – both maximal and in average. Choosing from simpler models, for short-range forecasts (1-3 years) linear extrapolation of population size can be used, but for horizons longer than five model (I) is better than extrapolation, and only model (II) is better than (I). Models proposed, being aggregate and adequate to real population reproduction as well, can be used in both applied estimations and theoretical research of the population, in construction of other demographic and economic-demographic models.

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